

# REPRESENTATION THEORY OF FINITE GROUPS

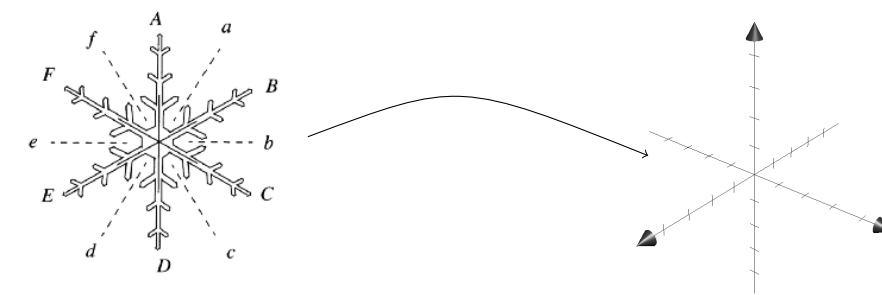
Spring 2024

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<b>Time:</b> TBD	<b>Place:</b> MSB: Room TBD

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**Description:** This seminar will be an introduction to representation theory. If you like group theory and linear algebra this is the perfect topic for you! Representation theory tells us how to write a group as a group of matrices and is an essential tool for understanding groups better. We will use knowledge of finite groups and invertible matrices to understand how we can encode the action of a group element on a vector space in a matrix.



**Prerequisites:** MAT 150A or equivalent group theory course, and familiarity with linear algebra.

## Texts:

- [The Symmetric Group](#) (Sagan), Chapter 1 and exercises.
- [Representations and Character Groups](#) (James, Liebeck) Ch 1-14 for supplementary examples and exercises.

## Learning Objectives:

1. Students will learn what a group representation is and their relationship to modules.
2. Students will learn main theorems about reducibility of representations such as Maschke's Theorem.
3. Students will be able to construct character tables for the symmetric group.

## Meeting Expectations:

- Students will be expected to present every other week on main results from the week's reading (proofs may be omitted) and/or 1-2 exercises. Or they may come with questions about the material for everyone to discuss together.
- If time permits, students will break into smaller groups to discuss the material.
- All students are expected to read the week's material even if they are not presenting.

- Students are NOT expected to master the material in one week and are encouraged to ask questions.
- At the end of each meeting students will be given 1-2 exercises to think about and discuss the following week. Students do not need to have a completed solution but should at least come with ideas or questions to discuss with peers.

**Course Outline:** Pacing may be modified according to how the students are doing, each meeting starts with a check in on how reading and exercises are going.

\*Exercises will be given according to students' level of understanding.

- Meeting 1:
  - Introductions
  - Review of group theory and motivation for representation theory
  - Liebeck Ch 3 Exercises
- Meeting 2:
  - Students present Sagan section 1.2 (Matrix Representations)
  - Students present Liebeck Ch3 Exercises
  - Exercises tbd
- Meeting 3:
  - Lecture on Sagan 1.3 (G-Modules and the Group Algebra)
  - Exercises on modules and group algebras (Liebeck Ch4 tbd)
- Meeting 4:
  - Students present Sagan sections 1.4, 1.5 (Reducibility, Maschke's Theorem)
  - Exercises tbd
- Meeting 5:
  - Lecture on Sagan section 1.6 (G-Homomorphisms and Schur's Lemma)
  - Exercises tbd
- Meeting 6:
  - Students present Sagan Thrm 1.7.8, Thrm 1.7.9, and Section 1.8 (Commutant and Endomorphism Algebras, Group Characters)
  - Exercises tbd
- Meeting 7:
  - Lecture on Section 1.9 (Inner Products of Characters)
  - Examples on constructing character tables
  - Exercises tbd
- Meeting 8:
  - Students present Section 1.10 (Decomposition of the Group Algebra)
  - Exercises tbd
- Meeting 9 (Optional):
  - Recap and activity/example of an application of Representation Theory
  - Discuss exercises